

IERG5154 Homework 1

Policy: You may verbally discuss problems, and your attempted solutions, with each other. However, you're not allowed to show each other your solutions. In particular, you're not allowed to read anything any of your classmates writes.

1. *Standard deviation of expected description length (5 points)* In the coding scheme in worksheet 1, we showed only that the expected description length of the source code was approximately $nH(X)$. Prove that an upper bound on the standard deviation of this length is $c\sqrt{n}$ for some constant c .
2. *Source coding with side information (5 points)* Two random variables (X, Y) are jointly distributed i.i.d according to a distribution $q(x, y)$. Xavier has access to the sequence of pairs $\{(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)\}$, and Yvonne has access only to $\{Y_1, Y_2, \dots, Y_n\}$. Describe a coding scheme that allows Xavier to describe the sequence $\{X_1, X_2, \dots, X_n\}$ to Yvonne using only approximately $nH(X|Y)$ bits.
3. *"Triple" mutual information (5 points)* As discussed in class, the quantity $I(X; Y) - I(X; Y|Z)$ (corresponding to the centre of the Venn diagram we drew in class) can be either positive or negative. Construct two probability distributions - one for each case.
4. *Csiszar sum inequality (5 points)* Prove that for any sequence of random variables $\{(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)\}$,

$$\sum_{i=1}^n I(X_{i+1}^n; Y_i | Y_1^{i-1}) = \sum_{i=1}^n I(Y_1^{i-1}; X_i | X_{i+1}^n).$$

Here X_a^b denotes the sequence $\{X_a, X_{a+1}, \dots, X_b\}$ (similarly for Y).

5. (0 points) Why did the chicken cross the road?